

Rigorous Effective Field Theory Study on Pion Form Factor

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We study $e^+e^- \rightarrow \pi^+\pi^-$ cross section and phase shift of $I = l = 1$ $\pi - \pi$ scattering below 1GeV in framework of chiral constituent quark model. The results including all order contribution of the chiral perturbation expansion and all one-loop effects of pseudoscalar mesons, but without any adjust parameters. Width of ρ predicted by the model strongly depends on transition momentum-square q^2 . We show that the mass parameter of ρ -meson in its propagator is very different from its physical mass due to momentum-dependent width of ρ . The mass difference between ρ^0 and ω are predicted successfully. The rigorous theoretical prediction on $e^+e^- \rightarrow \pi^+\pi^-$ cross section and the phase shift in $I = l = 1$ $\pi - \pi$ scattering agree with data excellently.

32.80.Cy,12.40.Vv,12.39.Fe,12.40.Yx

The process $e^+e^- \rightarrow \pi^+\pi^-$ at energies lower than the chiral symmetry spontaneously breaking scale contains very important information on low energy hadron dynamics. It was an active subject and studied continually during past fifty years. Experimentally, the effects of the strong interaction in process of e^+e^- annihilation is obvious to provides a large enhancement to production of pions in vector meson resonance region[1-5]. Theoretically, however, the problem was not studied by using a rigorous effective field theory(EFT) of QCD yet. Although at very low energy the chiral perturbative theory(ChPT) is a rigorous EFT of QCD, it in principle can not predict physics at vector meson resonance region. From viewpoint of quantum field theory, a rigorous theoretical study on $e^+e^- \rightarrow \pi^+\pi^-$ cross section and $l = 1$, $I = 1$ $\pi - \pi$ phase shift provided by an EFT of QCD must satisfy the following requirements: 1) Some fundamental principles, such as symmetry and unitarity, must be satisfied in this EFT. 2) The experimental data of $l = 1$, $I = 1$ $\pi - \pi$ scattering phase shift implies that width of ρ -meson Γ_ρ is transitional momentum-dependent, and must vanish at $q^2 = 0$ (where q^2 denotes four-momenta square of off-shell ρ). The momentum-dependence of $\Gamma_\rho(q^2)$ should be predicted by the EFT itself instead of being fitted by experiment. 3) This EFT must provide an effective method to evaluated error bar of this current calculation, i.e., the next order contribution should be able to be calculated. The purpose of this present paper is to provide a rigorous EFT study on pion form factor and $I = 1$, P-wave $\pi\pi$ phase shift below 1GeV. In other words, all requirements mentioned above will be met in the study of this paper.

In some recent references [6–8], the authors have studied $e^+e^- \rightarrow \pi^+\pi^-$ cross section and $l = 1$, $I = 1$ phase shift at vector meson resonance region by using some very simple phenomenological models. These models are constructed in the intermediate energy region using some phenomenology considerations, such as vector meson dominant(VMD) and universal coupling. In principle, each of them can capture some leading order effects of low energy EFT of QCD and they are classified by different symmetry realization for vector meson fields [9]. However, so far, the low energy effective lagrangians including vector meson resonances are only up to $O(p^4)$ which are not enough for the physics at vector meson mass scale, and can not successfully evaluate very important one-loop effects of pseudoscalar mesons which corresponds to the next to leading order of N_c^{-1} expansion. Hence, these phenomenological models are not of rigorous EFT, and they can not provide any rigorous theoretical predictions on low energy hadron physics. This

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bad shortage can be overcome by using the EFT in ref. [10], in which we constructed a consistent chiral constituent quark model(ChCQM) with lowest vector meson resonances and element Goldstone bosons. In this formalism we can capture all order information of chiral perturbative expansion and one-loop effects of pseudoscalar mesons.

In chiral limit, ChCQM is parameterized by the following chiral constituent quark lagrangian

$$\mathcal{L}_\chi = i\bar{q}(\not{\partial} + \not{F} + g_A \not{A} \gamma_5 - i\not{V})q - m\bar{q}q + \frac{F^2}{16} \langle \nabla_\mu U \nabla^\mu U^\dagger \rangle + \frac{1}{4} m_0^2 \langle V_\mu V^\mu \rangle. \quad (1)$$

Here $\langle \dots \rangle$ denotes trace in SU(3) flavour space, $\bar{q} = (\bar{q}_u, \bar{q}_d, \bar{q}_s)$ are constituent quark fields. V_μ denotes vector meson octet and singlet,

$$V_\mu(x) = \lambda \cdot \mathbf{V}_\mu = \sqrt{2} \begin{pmatrix} \frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & \rho_\mu^+ & K_\mu^{*+} \\ \rho_\mu^- & -\frac{\rho_\mu^0}{\sqrt{2}} + \frac{\omega_\mu}{\sqrt{2}} & K_\mu^{*0} \\ K_\mu^{*-} & K_\mu^{*0} & \phi_\mu \end{pmatrix}. \quad (2)$$

The 3×3 anti-Hermitian matrices Δ_μ and Γ_μ are defined as follows,

$$\begin{aligned} \Delta_\mu &= \frac{1}{2} \{ \xi^\dagger (\partial_\mu - ir_\mu) \xi - \xi (\partial_\mu - il_\mu) \xi^\dagger \}, \\ \Gamma_\mu &= \frac{1}{2} \{ \xi^\dagger (\partial_\mu - ir_\mu) \xi + \xi (\partial_\mu - il_\mu) \xi^\dagger \}, \end{aligned} \quad (3)$$

and covariant derivative are defined as follows

$$\begin{aligned} \nabla_\mu U &= \partial_\mu U - ir_\mu U + iU l_\mu = 2\xi \Delta_\mu \xi, \\ \nabla_\mu U^\dagger &= \partial_\mu U^\dagger - il_\mu U^\dagger + iU^\dagger r_\mu = -2\xi^\dagger \Delta_\mu \xi^\dagger, \end{aligned} \quad (4)$$

where $l_\mu = v_\mu + a_\mu$ and $r_\mu = v_\mu - a_\mu$ are linear combinations of external vector field v_μ and axial-vector field a_μ , ξ associates with non-linear realization of spontaneously broken global chiral symmetry introduced by Weinberg [11]. This realization is obtained by specifying the action of global chiral group $G = SU(3)_L \times SU(3)_R$ on element $\xi(\Phi)$ of the coset space $G/SU(3)_V$:

$$\xi(\Phi) \rightarrow g_L \xi(\Phi) h^\dagger(\Phi) = h(\Phi) \xi(\Phi) g_L^\dagger, \quad g_L, g_R \in G, \quad h(\Phi) \in H = SU(3)_V. \quad (5)$$

Explicit form of $\xi(\Phi)$ is usual taken

$$\xi(\Phi) = \exp \{ i\lambda^a \Phi^a(x)/2 \}, \quad U(\Phi) = \xi^2(\Phi), \quad (6)$$

where the Goldstone boson Φ^a are treated as pseudoscalar meson octet. In ref. [10] we have shown that the lagrangian (1) is invariant under $G_{\text{global}} \times G_{\text{local}}$. The axial coupling constant $g_A = 0.75$ is fitted by β -decay of neutron, and constituent quark mass $m = 480\text{MeV}$ is fitted by low energy limit of the model. It has been also illustrated that the value of g_A has included effects of intermediate axial-vector meson resonances exchanges at low energy.

The EFT describing low energy meson interaction can be deduced via loop effects of constituent quarks [10]. From viewpoint of symmetry, at leading order of vector mesons coupling to pseudoscalar mesons, the effective lagrangian is equivalent to WCCWZ lagrangian given by Brise [9,13]. In terms of this EFT, we found that the chiral perturbative expansion converge slowly at vector meson energy scale. Thus the high order contributions of chiral perturbative expansion play important role at this energy scale. Phenomenologically, this model provides excellent theoretical predictions on ρ -physics [10] and on ω -physics [12]. In this present paper, we focus our attention on vector-photon, vector- $\pi\pi$ and photon- $\pi\pi$ vertices. These relevant vertices have been calculated in ref. [10,12] which including all order effects of the chiral perturbative expansion and one-loop contribution of pseudoscalar mesons. The ‘‘direct’’ photon- $\pi\pi$ coupling and vector-photon coupling vertices read,

$$\begin{aligned} \mathcal{L}_{\gamma\pi\pi}^c &= \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} \bar{F}_\pi(q^2) A_\mu(q) [\pi^+(x) \partial^\mu \pi^-(x) - \partial^\mu \pi^+(x) \pi^-(x)], \\ \mathcal{L}_{\rho\gamma}^c &= -\frac{1}{2} e \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} b_{\rho\gamma}(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \rho^{0\mu}(q) A^\nu(x), \\ \mathcal{L}_{\omega\gamma}^c &= -\frac{1}{6} e \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} b_{\rho\gamma}(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) A^\nu(x), \end{aligned} \quad (7)$$

where the super-script “c” denotes these “complete” effective couplings which have contained one-loop effects of pseudoscalar mesons so that the “form factors”, $\bar{F}_\pi(q^2)$, $b_{\rho\gamma}(q^2)$ etc., are not real function. These “form factors” are given as follows [10,12],

$$\begin{aligned}
\bar{F}_\pi(q^2) &= 1 + \frac{q^2 b_\gamma(q^2)}{1 + \Sigma(q^2)}, & b_{\rho\gamma}(q^2) &= \frac{A(q^2)}{g(1 + \zeta)} - f_\pi^2 b(q^2) \frac{\Sigma_0(q^2)}{1 + 2\zeta} \left[1 + \frac{q^2 b_\gamma(q^2)}{1 + \Sigma(q^2)} \right], \\
b_\gamma(q^2) &= \frac{gb(q^2)}{2(1 + 3\zeta)} - \frac{1}{16\pi^2 f_\pi^2} \left\{ \lambda + \int_0^1 dx \cdot x(1-x) \ln \left[\left(1 - \frac{x(1-x)p^2}{m_\kappa^2} \right) \left(\frac{x(1-x)p^2}{m_\kappa^2} \right)^2 \right] \right. \\
&\quad \left. - \frac{2}{3} i\pi\theta(p^2 - 4m_\pi^2) \right\} - \frac{C(q^2)\Sigma_0(q^2)}{(1 + 11\zeta/3)}, \\
b(q^2) &= \frac{1}{gf_\pi^2} [A(q^2) + g_A^2 B(q^2)], & C(q^2) &= \frac{A(q^2) + 2g_A^2 B(q^2)}{2f_\pi^2}, \\
A(q^2) &= g^2 - \frac{N_c}{\pi^2} \int_0^1 dt \cdot t(1-t) \ln \left(1 - \frac{t(1-t)q^2}{m^2} \right), \\
B(q^2) &= -g^2 + \frac{N_c}{2\pi^2} \int_0^1 dt_1 \cdot t_1 \int_0^1 dt_2 (1 - t_1 t_2) \left[1 + \frac{m^2}{m^2 - t_1(1-t_1)(1-t_2)q^2} \right. \\
&\quad \left. + \ln \left(1 - \frac{t_1(1-t_1)(1-t_2)q^2}{m^2} \right) \right], \\
\Sigma_0(q^2) &= \frac{2}{f_\pi^2} [2\Sigma_\pi(q^2) - \Sigma_K(q^2)], & \Sigma(q^2) &= \left[1 + \frac{q^2 C(q^2)}{1 + 11\zeta/3} \right] \Sigma_0(q^2), \\
\Sigma_K(q^2) &= \frac{1}{(4\pi)^2} \left\{ \lambda(m_\kappa^2 - \frac{q^2}{6}) + \int_0^1 dt [m_\kappa^2 - t(1-t)q^2] \ln \left(1 - \frac{t(1-t)q^2}{m_\kappa^2} \right) \right\}, \\
\Sigma_\pi(q^2) &= \frac{q^2}{(4\pi)^2} \left\{ \frac{\lambda}{6} + \int_0^1 dt \cdot t(1-t) \ln \frac{t(1-t)q^2}{m_\kappa^2} - \frac{i}{6} \pi\theta(q^2 - 4m_\pi^2) \right\},
\end{aligned} \tag{8}$$

where $f_\pi = 185\text{MeV}$ is decay constant of pion, g and λ are constants which absorb the logarithmic divergence from constituent quark loops and the quadratic divergence from meson loops respectively,

$$\begin{aligned}
g^2 &= \frac{8}{3} \frac{N_c}{(4\pi)^{D/2}} \left(\frac{\mu^2}{m^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}), \\
\lambda &= \left(\frac{4\pi\mu^2}{m_\kappa^2} \right)^{\epsilon/2} \Gamma(2 - \frac{D}{2}), & \zeta &= \frac{2\lambda}{(4\pi)^2} \frac{m_\kappa^2}{f_\pi^2}.
\end{aligned} \tag{9}$$

In ref. [10], $\lambda = 2/3$ has been fitted by Zweig rule.

Traditionally, VMD [14] assumes that all photon-hadron coupling is mediated by vector mesons. However, from an empirical or symmetrical point of view, one has a freedom, that a non-resonant background is allowed. For instance, in process of $e^+e^- \rightarrow \pi^+\pi^-$, since $\pi^+\pi^-$ can consist of a vector-isovector system whose quantum numbers are same to ρ^0 , experiment or symmetry can not divide contribution of “direct” photon- $\pi\pi$ coupling from one from photon $\rightarrow \rho^0 \rightarrow \pi^+\pi^-$. Therefore, in general, the traditional VMD is a strong assumption. From eq. (7), we can see that the “direct” photon- $\pi\pi$ coupling indeed exists in this EFT. The same problem is also questioned in isospin breaking decay $\omega \rightarrow \pi^+\pi^-$, which is dominated by ρ^0 exchange, but have a contribution from “direct” $\omega\pi^+\pi^-$ coupling yet.

The “complete” $\rho\pi\pi$ vertex reads

$$\mathcal{L}_{\rho\pi\pi}^c = \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} g_{\rho\pi\pi}(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \rho^{0\mu}(q) [\pi^+(x) \partial^\mu \pi^-(x) - \partial^\mu \pi^+(x) \pi^-(x)], \tag{10}$$

with

$$g_{\rho\pi\pi}(q^2) = \frac{b(q^2)}{(1 + 2\zeta)(1 + \Sigma(q^2))}. \tag{11}$$

At leading order of vector meson coupling, the VMD vertex and $\rho\pi\pi$ vertex read respectively

$$\begin{aligned}\mathcal{L}_{\rho\gamma}^c &= -\frac{1}{2}eg \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \rho^{0\mu}(q) A^\nu(x), \\ \mathcal{L}_{\rho\pi\pi}^c &= \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} \frac{1}{gf_\pi^2} [g^2 + g_A^2 (\frac{N_c}{3\pi^2} - g^2)] (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \rho^{0\mu}(q) [\pi^+(x) \partial^\mu \pi^-(x) - \partial^\mu \pi^+(x) \pi^-(x)].\end{aligned}\quad (12)$$

A gauge-like argument [14,15] suggests that the ρ couples to all hadrons with the same strength (universality). It is formulated by the first KSFR sum rule [16]

$$g_{\rho\gamma} = \frac{1}{2} f_\pi^2 g_{\rho\pi\pi}. \quad (13)$$

However, experimentally, the first KSFR sum rule is observed to be not quite exact [17]. It can be naturally understood the universal coupling is a conclusion only working at leading order of vector meson coupling, and high order contribution will correct it. From eq. (12) we can see that the first KSFR sum rule is strictly satisfied when $g = \pi^{-1}$ for $N_c = 3$. Thus $g = \pi^{-1}$ is a favorite choice. In addition, it has been shown in ref. [10] how high order correction breaks the first KSFR sum rules. Besides of the parameters g_A , m , g and λ , these are no other adjustable free parameters. So that this EFT will provide powerful prediction on low energy meson physics. For example, the theoretical prediction of on-shell decay width of $\rho^0 \rightarrow e^+e^-$ is 7.0MeV, which agree with experimental data, 6.77 ± 0.32 MeV, very well.

In this EFT, the ρ -resonance propagator (Breit-Wigner formula) can be naturally derived due to unitarity of the model instead of input [10],

$$\Delta_{\mu\nu}^{(\rho)}(q^2) = \frac{-i\delta_{\mu\nu}}{q^2 - \tilde{m}_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2)}, \quad (14)$$

where we have included only that part of the propagator which survives when coupled to conserved currents, \tilde{m}_ρ is the (real valued) **mass parameter** and $\Gamma_\rho(q^2)$ is the momentum-dependent width,

$$\Gamma_\rho(q^2) = -\frac{f_\pi^2 b^2(q^2) q^4}{2(1+2\zeta)^2} \sqrt{q^2} \text{Im}\left\{\frac{\Sigma_0(q^2)}{1+\Sigma(q^2)}\right\} = \frac{|g_{\rho\pi\pi}(q^2)|^2 q^4}{48\pi} \sqrt{q^2} \left(1 - \frac{4m_\pi^2}{q^2}\right)^{3/2}. \quad (15)$$

Numerically, the on-shell width $\Gamma_\rho = \Gamma_\rho(q^2 = m_\rho^2) = 146$ MeV, which agree with data very well.

Because the width in ρ -resonance (possessing a complex pole) propagator (14) is momentum-dependent, it must be addressed that **the mass parameter \tilde{m}_ρ is not the physical mass $m_\rho = 770$ MeV**. Let us interpret this point briefly. Empirically, the physical mass of resonance is defined as position of pole (real value) in relevant scattering cross section, or theoretically, it should be defined as real part of complex pole possessed by resonance. It is well-known that the width of ρ -resonance is generated by pion loops. For a simple VMD model, the leading order of $\rho - \pi\pi$ coupling is independent of q^2 . Thus one has $\Gamma_\rho^{(VMD)}(q^2) \propto \sqrt{q^2}$, and due to equation

$$q^2 - \tilde{m}_\rho^2 + i\frac{\Gamma_\rho}{m_\rho} q^2 = 0, \quad (16)$$

we obtain the complex pole of ρ -resonance is $q^2 = m_\rho^2(1 - i\epsilon + O(\epsilon^2))$ with $\epsilon = \Gamma_\rho/m_\rho \simeq 0.19$. The result yields $\tilde{m}_\rho = m_\rho \sqrt{1 + \epsilon^2} = 784$ MeV. In particular, in the EFT used by this present paper, $\rho - \pi\pi$ coupling is proportional to q^2 at least. Hence one has $\Gamma_\rho(q^2) \propto q^4 \sqrt{q^2}$ at least, and complex pole equation

$$q^2 - \tilde{m}_\rho^2 + i\frac{\Gamma_\rho}{m_\rho^5} q^6 = 0. \quad (17)$$

It yields $\tilde{m}_\rho = m_\rho \sqrt{1 + 3\epsilon^2} = 810$ MeV, which poses a significant correction.

The above discussions imply that: 1) For resonance with large width, the mass parameter in its propagator is different from its physical mass. The correction is proportional to the ratio of resonant width to physical mass. 2) The mass in the original effective lagrangian only emerges as a parameter instead of a physical quantity measured directly in experiment. 3) The choice of mass parameter is relied on the choice of model. But the physical quantity must be independent of this choice.

Since in our result all hadronic couplings include all order information of the chiral perturbative expansion and one-loop effects of pseudoscalar mesons, the momentum-dependence of $\Gamma_\rho(q^2)$ is very complicate. It is difficult to determine \tilde{m}_ρ via the above method. Note that it is welcome that all vector meson resonances degenerate into a universal mass parameter m_v at chiral limit and large N_c limit. A reliable method is to determine m_v via input mass

of ω -resonance (since $\Gamma_\omega \ll m_\omega$, \tilde{m}_ω is almost equal to m_ω and hereafter we do not distinguish them). Then \tilde{m}_ρ can be obtained via dynamical calculation provide by this EFT. In general, the splitting between \tilde{m}_{ρ^0} and m_ω is caused by three sources: $\rho^0 - \omega$ mixing, electromagnetic effects due to VMD and one-loop effects of pseudoscalar mesons. Up to next to leading order N_c^{-1} expansion, the momentum-dependent $\rho^0 - \omega$ mixing has been derived in ref. [12],

$$\mathcal{L}_{\omega\rho}^c = \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} \Theta_{\omega\rho}(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \rho^{0\nu}(x), \quad (18)$$

where

$$\begin{aligned} \Theta_{\omega\rho}(q^2) = & \frac{N_c}{6\pi^2} \frac{m_u - m_d}{m} \left\{ g^{-2} h_0(q^2) \left(1 - \frac{4}{3}\zeta\right) + q^2 b(q^2) s(q^2) [\Sigma_K(q^2) - \frac{\Sigma_\pi(q^2)}{1 + \Xi(q^2)}] \right\} \\ & + \frac{\alpha_{\text{e.m.}} \pi}{3} b_{\rho\gamma}^2(q^2) + O((a_1(m_u - m_d) + a_2 \alpha_{\text{e.m.}})^2). \end{aligned} \quad (19)$$

The function $b(q^2)$, $b_{\rho\gamma}(q^2)$, $\Sigma_K(q^2)$ and $\Sigma_\pi(q^2)$ are given in eq.(8), and $s(q^2)$, $h_0(q^2)$ and $\Xi(q^2)$ are of follows

$$\begin{aligned} s(q^2) &= \frac{4}{gf_\pi^2} [h_0(q^2) + \frac{3}{4} g_A^2 (h_1(q^2) - \frac{h_2(q^2)}{2})], \\ h_0(q^2) &= \int_0^1 dt \frac{6t(1-t)}{1 - t(1-t)q^2/m^2}, \\ h_1(q^2) &= \int_0^1 dt_1 \cdot t_1^2 \int_0^1 dt_2 (1-t_2) \frac{3 - 2t_1^2 t_2 (1+2t_1)(1-t_2)q^2/m^2}{[1 - t_1^2 t_2 (1-t_2)q^2/m^2]^2}, \\ h_2(q^2) &= \int_0^1 dt_1 \cdot t_1^2 \int_0^1 dt_2 (1-t_2) \frac{4(1-t_1)[3 - 4t_1^2 t_2 (1-t_2)q^2/m^2]}{[1 - t_1^2 t_2 (1-t_2)q^2/m^2]^2}, \\ \Xi(q^2) &= 4f_\pi^{-2} (1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2}) \Sigma_\pi(q^2). \end{aligned} \quad (20)$$

In addition, it should be also noticed that the π -loop correction to m_ω is suppressed by isospin conservation. Thus one-loop correction to m_ω is dominated by K -meson. At tree level, the $\omega - KK$ coupling reads

$$\begin{aligned} \mathcal{L}_{\omega KK} = & \frac{i}{2} \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} b(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) \\ & \times \{ [K^+(x) \partial^\nu K^-(x) - \partial^\nu K^+(x) K^-(x)] + [K^0(x) \partial^\nu \bar{K}^0(x) - \partial^\nu K^0(x) \bar{K}^0(x)] \}. \end{aligned} \quad (21)$$

Then the physical mass of ω -meson are

$$\begin{aligned} m_\omega^2 = & m_v^2 + \text{Re} \left\{ \frac{q^4 \Theta_{\omega\rho}(q^2)}{q^2 - \tilde{m}_\rho^2 + i\sqrt{q^2} \Gamma_\rho(q^2)} + \frac{\pi \alpha_{\text{e.m.}}}{9} q^2 b_{\rho\gamma}^2(q^2) \right\} \Big|_{q^2=m_\omega^2} \\ & - \frac{q^4 b^2(q^2) \Sigma_K(q^2)}{(1+2\zeta)^2 \{1 + 2f_\pi^{-2} [1 + \frac{q^2 C(q^2)}{1+11\zeta/3}] \Sigma_K(q^2)\}} \Big|_{q^2=m_\omega^2}. \end{aligned} \quad (22)$$

Input $m_\omega = 782\text{MeV}$, one has $m_v = 785.8\text{MeV}$. Similarly, the mass parameter of ρ -meson are

$$\tilde{m}_\rho^2 = m_v^2 + \text{Re} \left\{ \frac{q^4 \Theta_{\omega\rho}(q^2)}{q^2 - m_\omega^2 + im_\omega \Gamma_\omega} + \pi \alpha_{\text{e.m.}} q^2 b_{\rho\gamma}^2(q^2) + \frac{f_\pi^2 b^2(q^2) q^6 \Sigma_0(q^2)}{2(1+2\zeta)^2 (1 + \Sigma(q^2))} \right\} \Big|_{q^2=m_\rho^2}. \quad (23)$$

Using the above value of m_v , we have $\tilde{m}_\rho = 803.1\text{MeV}$ which is indeed significantly different from the physical mass $m_\rho = 770\text{MeV}$. Success of this prediction will be checked in the following by localizing the position of pole in cross section of $e^+e^- \rightarrow \pi^+\pi^-$. Furthermore, the detail calculation shows that, the $\rho^0 - \omega$ only makes \tilde{m}_ρ shift -0.25MeV , the VMD effects and one-loop effects of pseudoscalar mesons make \tilde{m}_ρ shift $+3.45\text{MeV}$ and $+14.1\text{MeV}$ respectively.

For working out full shape of $e^+e^- \rightarrow \pi^+\pi^-$ cross section, the $\omega - \pi\pi$ coupling is needed. It has been derived in ref. [12], in which not only the coupling via ρ exchange, but also the “direct” coupling are included,

$$\mathcal{L}_{\omega\pi\pi}^c = -i \int \frac{d^4q}{(2\pi)^4} e^{iq \cdot x} g_{\omega\pi\pi}(q^2) (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \omega^\mu(q) [\pi^+(x) \partial^\nu \pi^-(x) - \partial^\nu \pi^+(x) \pi^-(x)], \quad (24)$$

where

$$g_{\omega\pi\pi}(q^2) = \frac{q^2 \Theta_{\omega\rho} g_{\rho\pi\pi}(q^2)}{q^2 - \tilde{m}_\rho^2 + i\sqrt{q^2} \Gamma_\rho(q^2)} - g_{\omega\pi\pi}^{(0)}(q^2), \quad (25)$$

with “direct” coupling strength

$$g_{\omega\pi\pi}^{(0)}(q^2) = \frac{N_c}{12\pi^2} \frac{m_u - m_d}{m} \left\{ s(q^2) \left(\frac{1}{1 + \Xi(q^2)} - \frac{10}{3} \zeta \right) - 6f_\pi^{-2} \Sigma_K(q^2) [8m^2 f_\pi^{-2} b(q^2) - \frac{s(q^2)}{3} (1 + \frac{q^2 N_c}{4\pi^2 f_\pi^2})] \right\} + \frac{2\alpha_{\text{e.m.}} \pi}{3} \bar{F}_\pi(q^2) b_{\rho\gamma}(q^2). \quad (26)$$

Eqs. (7), (10) and (24) lead to the electromagnetic form factor of pion as follow

$$F_\pi(q^2) = 1 + \frac{q^2 b_\gamma(q^2)}{1 + \Sigma(q^2)} - \frac{q^4 b_{\rho\gamma}(q^2) g_{\rho\pi\pi}(q^2)}{2(q^2 - \tilde{m}_\rho^2 + i\sqrt{q^2} \Gamma_\rho(q^2))} - \frac{q^4 b_{\rho\gamma}(q^2) g_{\omega\pi\pi}(q^2)}{6(q^2 - m_\omega^2 + i\sqrt{q^2} \Gamma_\omega)}. \quad (27)$$

Here due to narrow width of ω , we ignore the momentum-dependence of Γ_ω . In this form factor, we can see that the contributions of resonance exchange accompany q^4 factor. Due to this reason, some authors declared that the pion form factor in WCCWZ EFT exhibits an unphysical high energy behaviour ($\mu > m_\rho$). However, this conclusion is wrong. It is caused by their wrong result for momentum-dependence of $\Gamma_\rho(q^2)$ which is fitted by experimental instead of by dynamical prediction. In fact, since $\sqrt{q^2} \Gamma_\rho(q^2)$ is proportional to q^6 at least, we do not need to worry that the form factor has a bad high energy behaviour. We can also see that there is a moment-dependent non-resonant contribution. It together with the contribution of resonance exchange determined the high energy behaviour of the factor. The cross-section for $e^+e^- \rightarrow \pi^+\pi^-$ is given by (neglecting the electron mass)

$$\sigma = \frac{\pi \alpha_{\text{e.m.}}^2}{3} \frac{(q^2 - 4m_\pi^2)^{3/2}}{(q^2)^{5/2}} |F_\pi(q^2)|^2. \quad (28)$$

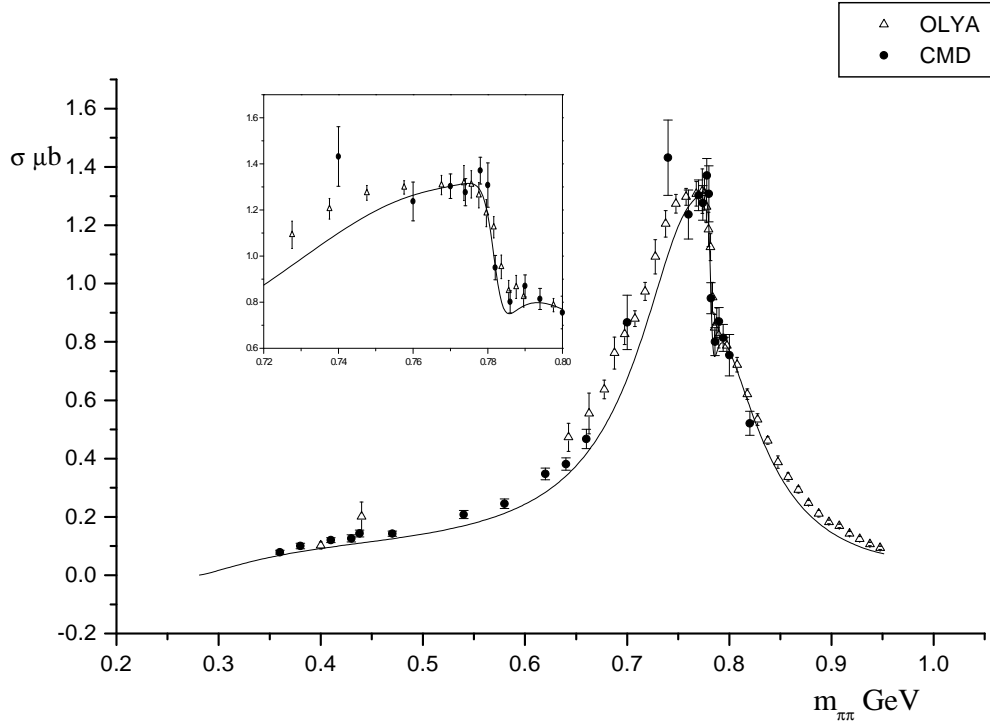


FIG. 1. $e^+e^- \rightarrow \pi^+\pi^-$ cross section. The experimental data are from refs.[1,2].

From definition of function $A(q^2)$ and $B(q^2)$ in eq. (8) we can see this EFT is unitary only for $q^2 < 4m^2$. Thus the effective prediction should be below $m_{\pi\pi} < 2m = 960\text{MeV}$. The result is shown in fig. 1. We can see the prediction agree with data well. Especially, the theoretical prediction in vector meson energy region agree with data excellently. Although the mass parameter $\tilde{m}_\rho = 803.1\text{MeV}$ in ρ propagator is larger than physical mass, the position of pole is localized in $\sqrt{q^2} = 772\text{MeV}$ which is just the physical mass of ρ . It strongly supports our above discussion and dynamical calculation. It also implies that we must carefully distinguish the physical mass difference of ρ^0 and ω from the difference of mass parameter in effective lagrangian.

Let us give some further remarks on pion form factor (27). From eqs. (8), (11) and (25) we can see that, in eq. (27), $b_\gamma(q^2)$, $b_{\rho\gamma}(q^2)$, etc., are all complex function instead of real function. It is caused by one-loop effects of pions. Thus the expression (27) can be rewritten as follow

$$F_\pi(q^2) = 1 + q^2 a_1(q^2) e^{i\phi_1(q^2)} - \frac{q^4 a_2(q^2) e^{i\phi_2(q^2)}}{2(q^2 - \tilde{m}_\rho^2 + i\sqrt{q^2}\Gamma_\rho(q^2))} - \frac{q^4 a_3(q^2) e^{i\phi_3(q^2)}}{6(q^2 - m_\omega^2 + i\sqrt{q^2}\Gamma_\omega)}. \quad (29)$$

Here $a_i(q^2)$ ($i = 1, 2, 3$) are three real function and $\phi_i(q^2)$ ($i = 1, 2, 3$) are three momentum-dependent phases. In particular, $\phi_3(q^2 = m_\omega^2)$, so called Orsay phase, has been extracted from data as $100 - 125$ degrees [7,8]. Our theoretical prediction is $\phi_3(q^2 = m_\omega^2) = 116.5$ degrees. However, so far, the phases $\phi_1(q^2)$ and $\phi_2(q^2)$ are not reported in any literatures. These momentum-dependent phases indicate that the dynamics including loop effects of pseudoscalar mesons is different from one only in tree level. In fig.2, we given theoretical curves of $\phi_i(q^2)$. They are indeed nontrivial.

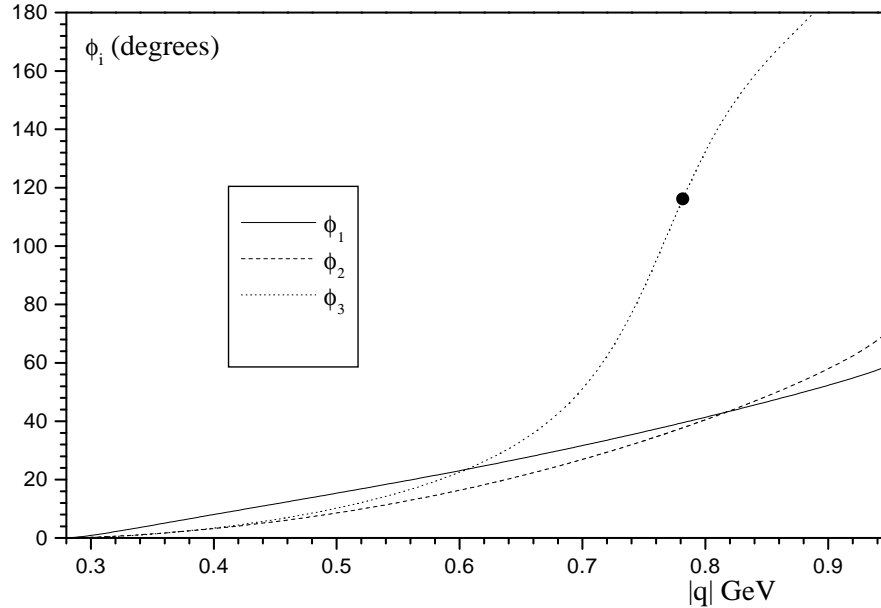


FIG. 2. ϕ_i versus $m_{\pi\pi}$ in GeV. Here the solide line denotes the phase shift of non-resonant background ϕ_1 , the dash line denotes the phase shift ϕ_2 in ρ coupling and the dot line denotes the phase shift ϕ_3 in ω coupling. “•” denotes Orsay phase.

Obviously, $F_\pi(q^2)$ is an analytic function in the complex q^2 plane, with a branch cut along the real axis beginning at the two-pion threshold, $q^2 = 4m_\pi^2$. Time-reversal invariance and the unitarity of the S -matrix requires that the phase of the form factor be that of $l = 1$, $I = 1$ $\pi - \pi$ scattering [18]. This last emerges as $\pi - \pi$ scattering in the relevant channel is very nearly elastic from threshold through $q^2 \simeq (m_\pi + m_\omega)^2$ [4,19]. In this region of q^2 , then, the form factor is related to the $l = 1$, $I = 1$ $\pi - \pi$ phase shift, δ_1^1 , via [20]

$$F_\pi(q^2) = e^{2i\delta_1^1(q^2)} F_\pi^*(q^2), \quad (30)$$

so that

$$\tan \delta_1^1(q^2) = \frac{\text{Im}F_\pi(q^2)}{\text{Re}F_\pi(q^2)}. \quad (31)$$

The above is a special case of what is sometimes called the Fermi-Watson-Aidzu phase theorem [20,21]. In fig. 3 and fig. 4 we plot theoretical curves of the $l = 1$, $I = 1$ $\pi - \pi$ phase shift δ_1^1 versus $m_{\pi\pi}$ and of $\sin \delta_1^1/p_\pi^3$ versus $m_{\pi\pi}$ (where $p_\pi = \frac{1}{2}\sqrt{q^2 - 4m_\pi^2}$) respectively. We omit the ω contribution from our plots of the phase of $F_\pi(q^2)$ for comparing with time-like region pion form factor data [22–24]. We have also assumed that δ_1^1 is purely elastic in the regime shown, i.e., the loop effects of $\omega - \pi$ are omitted. The curve predicts $\delta_1^1 \rightarrow 90^\circ$ as $\sqrt{q^2} \rightarrow 774\text{MeV} \simeq m_\rho$, and $\delta_1^1 > 100^\circ$ for $\sqrt{q^2} > 787\text{MeV}$. These results agree with data very well.

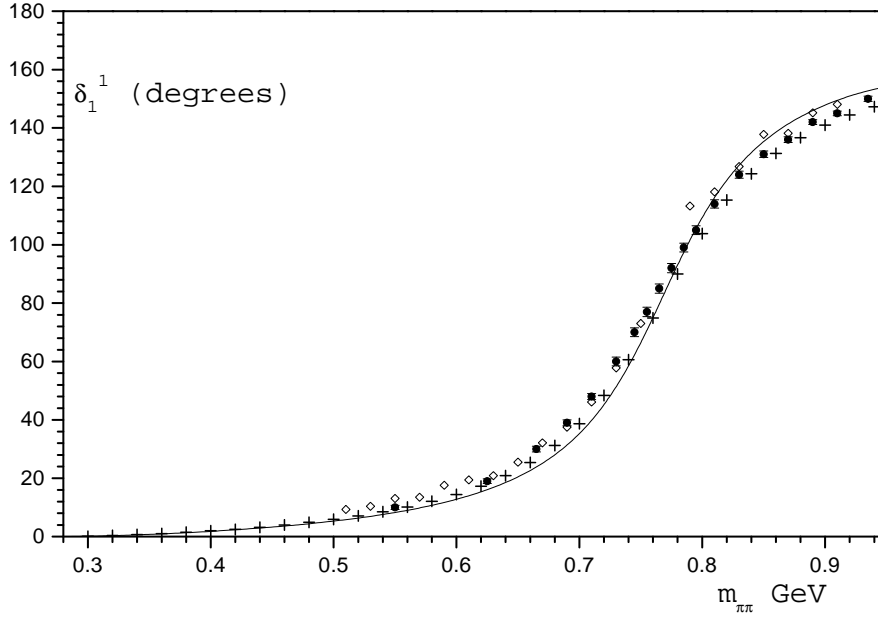


FIG. 3. The $l = 1$, $I = 1$ $\pi - \pi$ scattering phase shift. The solid circle point are these from [22], the hollow diamond point are from [23] and “+” denotes the points from [24]. Note that the ω contribution to the time-like pion form factor phase has been omitted, to facilitate comparison with the empirical phase shift.

Finally we discuss the near threshold behaviour of the form factor. 1) The chiral perturbative theory predicts the form factor at threshold to be $[F_\pi(4m_\pi^2)]_{\text{ChPT}} = 1.17 \pm 0.01$, and ours, $[F_\pi(4m_\pi^2)] = 1.154$, is close to the ChPT result. 2) The electromagnetic radius of charged pion has been determined to be $\sqrt{\langle r^2 \rangle_\pi} = 0.657 \pm 0.027\text{fm}$ [25], whereas the theoretical prediction in this present paper is $\sqrt{\langle r^2 \rangle_\pi} = 0.635\text{ fm}$. 3) The Froggatt-Petersen phase shift function $\sin \delta_1^1/p_\pi^3$ is connected with the vector-isovector $\pi - \pi$ scattering length a_1^1 through

$$a_1^1 = \lim_{q^2 \rightarrow 4m_\pi^2} \frac{\sin \delta_1^1}{p_\pi^3}. \quad (32)$$

Our theoretical prediction is $a_1^1 = 0.037$ in unit of m_π^{-3} . This value is very close to experimental results from K_{e4} data [26,27] using a Roy equation fit ($a_1^1 = 0.038 \pm 0.002$) and ChPT prediction $a_1^1 = 0.037 \pm 0.01$ [28] at the two loop order (at $O(p^4)$).

In summary, a rigorous, unitary EFT method has been applied to study pion form factor and $l = 1$, $I = 1$ $\pi - \pi$ phase shift. The theoretical predictions include all order informations of the chiral perturbative expansion and one-loop effects of pseudoscalar mesons. The Breit-Wigner formula for resonant propagators is derived by the EFT itself instead of an input. The momentum-dependence of $\Gamma_\rho(q^2)$ is predicted by the dynamics. It also has been revealed that

the mass parameter in resonant propagators should be different from its physical mass due to momentum-dependent width. This point is confirmed by both of dynamical calculation and phenomenological fit. It also tells us how to understand the mass splitting between ρ^0 and ω : Although the dynamical calculations show that the mass parameter of ρ in effective lagrangian is even larger than one of ω , the position of pole localized in real axis give their right physical mass splitting. The contribution to this mass splitting from $\rho^0 - \omega$ mixing is very small, the dominant contribution is from one-loops effects of pseudoscalar mesons. The EFT mechanics on $\rho^0 - \omega$ mass splitting revealed in the present paper is rather subtle. And it is another evident to confirm again that the EFT of QCD proposed in ref. [10] is sound. Actually, to the best of our knowledges, this is the first time to get $\rho^0 - \omega$ mass splitting through a well-defined quantum field theory calculation.

Due to one-loop effects of pions, the photon- $\pi\pi$, photon-vector and vector- $\pi\pi$ coupling are all with a nontrivial phase shift instead of purely real in some simple models. In a series of recent papers, we have revealed that the one-loop effects of pseudoscalar mesons play a very important role in low energy hadronic physics. Theoretically, it keeps unitarity of the S -matrix, and numerically, its contributions are about 30%. A well-defined EFT must be able to evaluate the high order contributions of the chiral perturbative expansion and N_c^{-1} expansion. It is an important criterion to judge a model as a rigorous EFT or a phenomenological model only.

In our study, no parameters need to be fitted by the data of pion form factor and $l = 1$, $I = 1$ $\pi - \pi$ phase shift. Thus our results are rigorous theoretical predictions and agree with data very well.

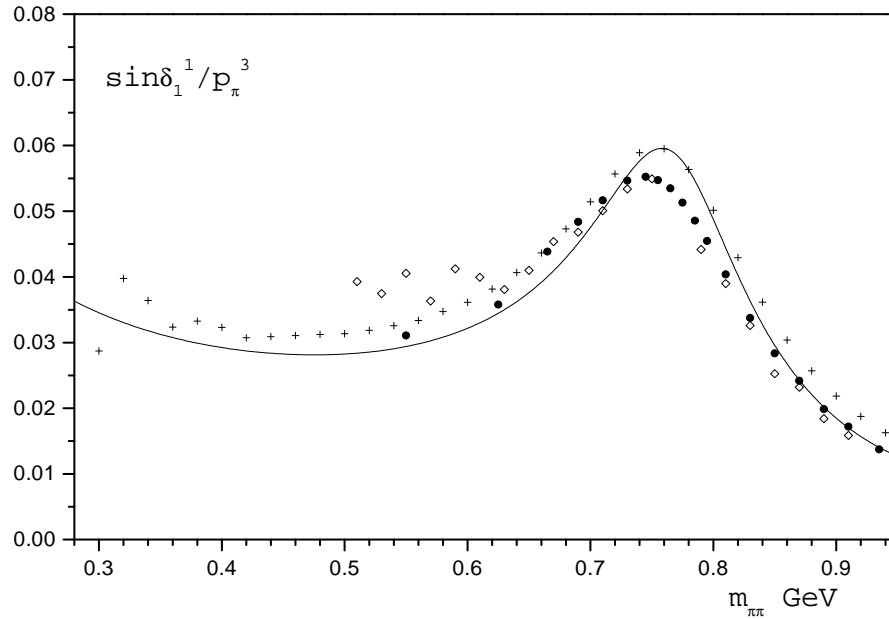


FIG. 4. Function $\sin \delta_1^1 / p_\pi^3$ deduced from $\pi^+\pi^-$ phase shift; the function is given in units of m_π^{-3} . The solid circle point are these from [22], the hollow diamond point are from [23] and “+” denotes the points from [24].

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